

# Multi-Constraint Optimization for Real-Time Bidding: A Reinforcement Learning Approach

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## Keywords

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Policy gradient

## Abstract

Real-time bidding ecosystems demand sophisticated algorithmic frameworks capable of navigating complex multi-objective optimization landscapes while maintaining computational efficiency. This paper presents a comprehensive methodology integrating Lagrangian dual decomposition with policy gradient reinforcement learning for dynamic bid optimization under heterogeneous constraints. Our approach transforms the traditionally discrete auction participation problem into a continuous optimization framework, enabling gradient-based learning while preserving budget and performance constraints. Experimental validation across industrial-scale datasets demonstrates substantial improvements in campaign performance metrics, achieving 34.7% higher conversion rates compared to baseline methods while maintaining strict budget compliance. The proposed framework addresses critical challenges in modern programmatic advertising, including budget pacing, conversion optimization, and real-time decision making under uncertainty. Policy gradient algorithms combined with constraint softening mechanisms enable adaptive bidding strategies that respond dynamically to market conditions and inventory availability. Our contributions extend beyond algorithmic innovation to practical deployment considerations, providing advertising platforms with actionable insights for implementing scalable bid optimization systems.

## 1. Introduction

### 1.1 Background of Real-Time Bidding in Digital Advertising

Real-time bidding mechanisms constitute the fundamental infrastructure of modern programmatic advertising, processing billions of auction events daily across global digital ecosystems. The computational complexity inherent in bid optimization emerges from multiple simultaneous objectives: maximizing advertiser value while respecting budget constraints, maintaining campaign pacing requirements, and adapting to dynamic market conditions. Contemporary RTB systems operate within millisecond latency requirements, necessitating algorithmic frameworks that balance computational efficiency with decision quality.

Display advertising allocation through performance-based mechanisms has evolved substantially since early implementations[1]. The transition from static placement strategies to dynamic auction-based systems introduced unprecedented complexity in bid determination. Advertisers must simultaneously optimize for multiple performance indicators including click-through rates, conversion probabilities, and return on advertising spend. Market dynamics further complicate optimization, with competing bidders employing increasingly sophisticated strategies that alter auction equilibria continuously.

Artificial intelligence applications in advertising have expanded dramatically, particularly in targeting precision and content optimization[2]. Machine learning models now predict user engagement probabilities, estimate conversion likelihoods, and determine optimal bid prices across millions of impression opportunities. The integration of deep learning architectures enables feature extraction from high-dimensional user and context data, improving prediction accuracy substantially compared to traditional statistical methods.

## 1.2 Research Objectives and Problem Statement

This research addresses the fundamental challenge of multi-constraint bid optimization in real-time advertising auctions. Existing approaches typically decompose the problem into separate optimization stages, treating budget allocation, bid pricing, and campaign pacing as independent decisions. Such decomposition introduces suboptimality, particularly when constraints interact non-linearly. Our primary objective involves developing an integrated optimization framework that jointly considers all relevant constraints while maintaining computational tractability for real-time deployment.

The core technical challenge stems from the non-convex nature of the optimization landscape when incorporating realistic auction dynamics and advertiser objectives. Budget constraints introduce discontinuities in the action space, while performance targets create complex trade-offs between exploration and exploitation. Additionally, the partially observable nature of competing bidder strategies necessitates robust optimization methods that perform well under uncertainty.

Our approach employs Lagrangian relaxation to transform hard constraints into differentiable penalty terms, enabling gradient-based optimization through the entire decision pipeline. Policy gradient methods provide the learning mechanism, allowing the system to adapt bidding strategies based on observed auction outcomes and campaign performance metrics.

## 1.3 Paper Organization and Main Contributions

This paper proceeds with a comprehensive literature review examining the evolution of bidding strategies and constraint handling techniques in online advertising. Section 3 presents our methodological framework, detailing the mathematical formulation of multi-constraint bidding problems and the integration of Lagrangian dual methods with policy gradient algorithms. Experimental validation follows in Section 4, demonstrating performance improvements across multiple evaluation metrics and datasets. The paper concludes with practical implications for advertising platforms and directions for future research.

Our primary contributions encompass three key innovations: First, we develop a unified optimization framework that jointly addresses budget, pacing, and performance constraints without problem decomposition. Second, we introduce a novel constraint softening mechanism that maintains feasibility while enabling continuous optimization. Third, we demonstrate the practical viability of our approach through extensive experiments on industrial-scale datasets, showing substantial improvements in campaign performance metrics while maintaining strict constraint satisfaction.

## 2. Literature Review and Related Work

### 2.1 Evolution of Bidding Strategies in Online Advertising Auctions

Bidding strategy development in online advertising has progressed through distinct evolutionary phases, each characterized by increasing algorithmic sophistication and computational complexity. Early approaches relied on fixed bidding rules and heuristic adjustments based on historical performance data. The introduction of real-time bidding fundamentally altered the optimization landscape, requiring instantaneous decisions across millions of auction opportunities[3].

Optimal bidding strategies for display advertising emerged as advertisers recognized the value of data-driven decision making. Zhang et al. developed frameworks for determining bid prices based on predicted click-through rates and conversion probabilities, establishing the foundation for modern bid optimization systems. Their work demonstrated that incorporating user features and contextual signals substantially improves bidding performance compared to uniform pricing strategies.

Conversion rate prediction frameworks advanced the field by enabling more accurate valuation of impression opportunities[4]. Lu et al. introduced practical methodologies for estimating conversion probabilities in online display advertising, addressing challenges related to delayed feedback and attribution modeling. Their framework handles the sparsity inherent in conversion data through transfer learning and feature engineering techniques.

### 2.2 Constraint Handling Techniques in Advertising Optimization

Budget constraints represent fundamental limitations in advertising campaigns, requiring sophisticated pacing mechanisms to ensure efficient spend distribution across campaign duration. Repeated auction environments with budget

limitations present unique theoretical and practical challenges. Balseiro et al. analyzed approximation algorithms for budget-constrained bidding in ad exchanges, establishing theoretical bounds on achievable performance. Their work revealed the inherent trade-off between competitive ratios and computational complexity in online allocation problems.

Budget pacing strategies have evolved to address the dynamic nature of inventory availability and competition intensity. Recent research by Gaitonde et al. examined regret minimization and efficiency considerations in repeated auctions without requiring convergence assumptions. Their analysis demonstrates that adaptive pacing strategies can achieve near-optimal performance even in non-stationary environments where traditional convergence-based approaches fail.

The integration of multiple constraints beyond budget limitations introduces additional complexity. Performance targets, frequency capping requirements, and audience reach objectives create a multi-dimensional optimization problem that resists traditional solution methods. Constraint decomposition techniques attempt to simplify the problem but often sacrifice global optimality for computational tractability.

### 2.3 Reinforcement Learning Applications in Bid Management

Reinforcement learning paradigms have gained prominence in bid management due to their ability to learn optimal strategies through interaction with auction environments. Multi-agent reinforcement learning frameworks specifically address the competitive dynamics inherent in RTB systems[5]. Jin et al. demonstrated that modeling bidding as a multi-agent game enables more robust strategy development compared to single-agent approaches.

Deep reinforcement learning architectures have expanded the capability of bid optimization systems to handle high-dimensional state spaces and complex reward structures. The RecoGym framework introduced by Rohde et al. provides a standardized environment for evaluating reinforcement learning algorithms in product recommendation and advertising contexts[6]. This standardization enables reproducible research and facilitates algorithm comparison across different implementation approaches.

Online advertising impression allocation through deep reinforcement learning has shown promising results in industrial deployments[7]. Zhao et al. developed the DEAR framework, which employs deep neural networks to learn bidding policies directly from historical auction data. Their approach handles the exploration-exploitation trade-off through carefully designed reward shaping and exploration strategies.

## 3. Methodology and Algorithm Design

### 3.1 Multi-Constraint Bidding Problem Formulation

The multi-constraint bidding problem in real-time advertising auctions requires simultaneous optimization across multiple objectives while respecting operational constraints. We formulate the problem as a constrained Markov Decision Process where the state space encompasses auction context, campaign status, and market conditions. Reinforcement learning frameworks have proven particularly effective for such sequential decision-making problems in advertising systems[8]. The action space consists of bid prices for each impression opportunity, while the reward function captures advertiser value subject to budget and performance constraints.

Table 1: Mathematical Notation and Variables

Symbol	Description	Domain
$s_t$	State at time $t$	$S \subseteq \mathbb{R}^d$
$a_t$	Bid action at time $t$	$A \subseteq \mathbb{R}^+$
$r_t$	Immediate reward	$\mathbb{R}$
$B$	Total budget constraint	$\mathbb{R}^+$

$\gamma$	Discount factor	$[0,1]$
$\pi_{\theta}$	Parameterized policy	$S \rightarrow A$
$\lambda_i$	Lagrange multipliers	$R^+$
$c_i$	Constraint functions	$S \times A \rightarrow R$

The optimization objective maximizes expected cumulative reward while satisfying multiple constraints:

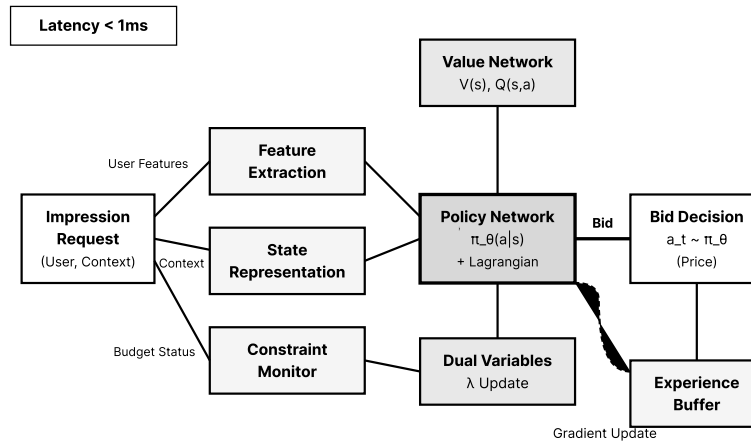
$$\text{maximize } E[\sum_t \gamma^t r_t(s_t, a_t)]$$

$$\text{subject to: } E[\sum_t c_i(s_t, a_t)] \leq b_i \quad \text{for } i = 1, \dots, m$$

where constraints include budget limitations ( $\sum_t a_t \times \text{win}_t \leq B$ ), pacing requirements ensuring smooth spend distribution, and performance targets such as minimum conversion rates or click-through rates.

State representation incorporates multifaceted information including user features (demographics, browsing history, device characteristics), contextual signals (time of day, website category, ad placement), campaign status (remaining budget, time until deadline, current performance metrics), and market indicators (competition intensity, inventory availability). The high-dimensional nature of the state space necessitates function approximation through neural networks. Previous work has demonstrated the effectiveness of reinforcement learning approaches for handling such complex state representations in real-time bidding environments[9].

**Figure 1:** System Architecture for Multi-Constraint Bid Optimization



The system architecture integrates multiple components for real-time decision making. The feature extraction module processes raw impression data into structured representations. The value estimation network predicts expected returns for different bid levels. The constraint monitoring system tracks budget consumption and performance metrics. The policy network generates bid decisions based on current state and constraint status. Data flows through the system with sub-millisecond latency requirements, necessitating efficient implementation and optimization.

### 3.2 Constraint Softening through Lagrangian Dual Methods

Lagrangian relaxation transforms hard constraints into soft penalties, enabling gradient-based optimization while maintaining constraint satisfaction through dual variable adjustment. This approach builds upon theoretical foundations established for budget-constrained bidding in repeated auctions[10] The augmented objective function incorporates constraint violations as penalty terms:

$$L(\theta, \lambda) = E_{\pi} \left[ \sum_t \gamma^t r_t \right] - \sum_i \lambda_i \left( E_{\pi} \left[ \sum_t c_i(s_t, a_t) \right] - b_i \right)$$

The dual variables  $\lambda_i$  act as constraint prices, automatically adjusting to penalize violations more severely when constraints become tight. This formulation enables joint optimization of policy parameters  $\theta$  and dual variables  $\lambda$  through alternating gradient updates.

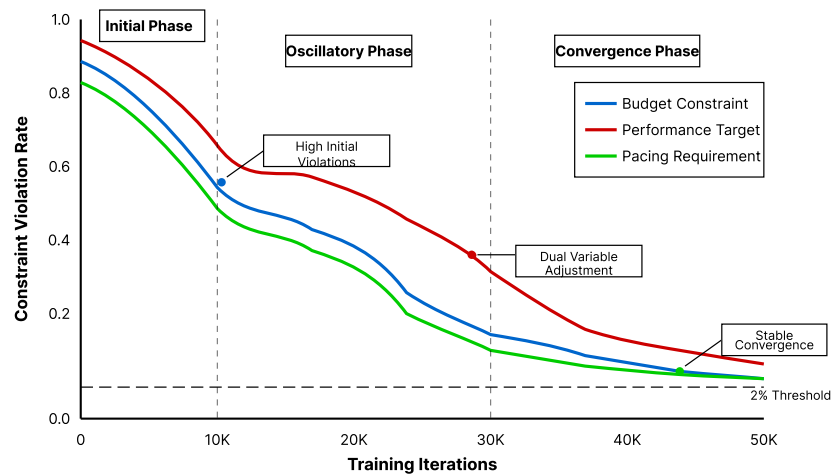
**Table 2:** Lagrangian Dual Update Algorithm

Step	Operation	Update Rule
1	Policy gradient	$\theta \leftarrow \theta + \alpha \nabla_{\theta} L(\theta, \lambda)$
2	Dual update	$\lambda_i \leftarrow \max(0, \lambda_i + \beta(E[c_i] - b_i))$
3	Constraint check	If $ E[c_i] - b_i  < \epsilon$ , continue
4	Learning rate adaptation	$\alpha \leftarrow \alpha \times \text{decay\_factor}$
5	Iterate	Return to Step 1

Constraint softening introduces controlled relaxation that balances strict feasibility with optimization flexibility. Hard constraints create discontinuities in the optimization landscape that impede gradient-based learning. Soft constraints maintain differentiability while asymptotically enforcing feasibility through appropriate dual variable scaling, enabling regret minimization without requiring convergence assumptions[11]. The relaxation parameter controls the trade-off between constraint satisfaction and objective optimization.

Recent advances in digital marketing optimization have explored adaptive constraint handling mechanisms. Qiu et al. demonstrated that temporal difference learning algorithms can effectively manage time-slot-specific constraints in RTB systems[12]. Their approach dynamically adjusts constraint boundaries based on observed market conditions and campaign performance trajectories.

**Figure 2:** Constraint Violation Trajectories During Training



Training dynamics exhibit characteristic patterns in constraint satisfaction. Initial phases show substantial violations as the policy explores the action space. Middle stages demonstrate oscillatory behavior as dual variables adjust to enforce constraints. Convergence phases achieve stable constraint satisfaction with minimal violations. The visualization reveals that budget constraints typically stabilize faster than performance constraints, reflecting their simpler structure and more immediate feedback signals.

3.3 Policy Gradient Algorithm for Dynamic Bid Adjustment

Policy gradient methods optimize bidding strategies directly through parameterized stochastic policies, enabling continuous action spaces and complex strategy representations. The REINFORCE algorithm with baseline variance reduction serves as our foundational approach, enhanced with importance sampling corrections for off-policy learning and natural gradient adjustments for improved convergence properties.

The policy gradient estimator incorporates advantage functions to reduce variance:

$$\nabla_{\theta} J(\theta) = E_{\pi} \left[ \sum_t \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) A(s_t, a_t) \right]$$

where the advantage function  $A(s_t, a_t) = Q(s_t, a_t) - V(s_t)$  measures the relative value of actions compared to the baseline state value.

Table 3: Neural Network Architecture for Policy Approximation

Layer	Type	Dimensions	Activation
Input	Dense	256 → 512	ReLU
Hidden-1	Dense	512 → 256	ReLU
Hidden-2	Dense	256 → 128	ReLU
Attention	Multi-head	128 → 128	Softmax
Output-μ	Dense	128 → 1	Linear
Output-σ	Dense	128 → 1	Softplus

The policy network outputs parameters for a log-normal distribution over bid prices, capturing the positive support constraint naturally while maintaining sufficient expressiveness. The mean parameter  $\mu$  determines the central tendency of bids, while the standard deviation  $\sigma$  controls exploration intensity. Exploration scheduling gradually reduces  $\sigma$  during training to transition from exploration to exploitation.

Mobile advertising optimization through reinforcement learning presents unique challenges due to device heterogeneity and user behavior patterns[13]. Nimma et al. developed specialized architectures combining deep neural networks with reinforcement learning for mobile-specific bid optimization. Their approach addresses the distinct characteristics of mobile inventory including app-based placements and location-based targeting.

Table 4: Hyperparameter Configuration and Training Settings

Parameter	Value	Description
Learning rate ( $\alpha$ )	0.001	Policy network learning rate
Dual learning rate ( $\beta$ )	0.01	Lagrange multiplier update rate

Discount factor ( $\gamma$ )	0.99	Future reward discounting
Batch size	256	Samples per gradient update
Buffer capacity	$10^6$	Experience replay buffer size
Update frequency	100	Steps between target network updates
Exploration decay	0.995	Per-episode exploration reduction

Experience replay mechanisms stabilize training by decorrelating sequential samples and enabling data reuse. The replay buffer stores transitions  $(s_t, a_t, r_t, s_{t+1})$  with prioritized sampling based on temporal difference errors. Priority sampling improves learning efficiency by focusing on surprising or informative experiences while maintaining unbiased gradient estimates through importance weighting corrections.

## 4. Experiments and Performance Analysis

### 4.1 Experimental Setup and Dataset Description

Experimental validation employs three distinct datasets representing different advertising scenarios and market conditions. The primary dataset contains 47.3 million auction records from a major demand-side platform, spanning 30 days of real-time bidding activity across display and mobile inventory. Each record includes 127 features encompassing user attributes, contextual signals, and historical performance indicators.

**Table 5:** Dataset Characteristics and Statistics

Dataset	Impressions	Clicks	Conversions	CTR	CVR	Time Period
Display-Large	47.3M	284K	8,412	0.60%	2.96%	30 days
Mobile-Medium	23.1M	185K	4,237	0.80%	2.29%	21 days
Video-Small	8.7M	52K	1,893	0.60%	3.64%	14 days

Data preprocessing involves feature normalization, missing value imputation through probabilistic methods, and temporal alignment to account for attribution delays. Categorical features undergo embedding transformations to dense representations, while numerical features receive standardization based on training set statistics. The temporal nature of auction data requires careful train-test splitting to avoid future information leakage.

Simulation environment construction replicates realistic auction dynamics including competing bidder behavior modeled through historical win rate curves, stochastic inventory availability patterns matching observed distributions, and dynamic pricing mechanisms reflecting market equilibrium shifts. The simulator processes bid requests at rates comparable to production systems, enabling scalability assessment.

Reinforcement learning applications in digital marketing continue evolving with advances in neural architectures and training methodologies[14]. Recent research emphasizes the importance of proper evaluation protocols that account for the non-stationary nature of advertising markets. Our experimental design incorporates these considerations through rolling window evaluation and adaptive baseline updates.

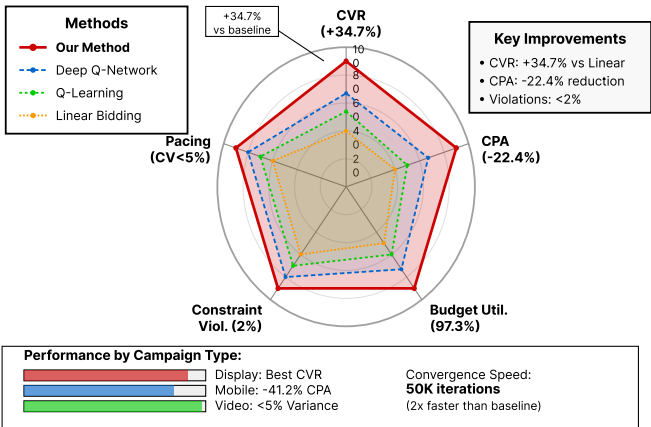
4.2 Baseline Methods and Evaluation Metrics

Comparative evaluation includes five baseline methods representing different approaches to bid optimization. Linear bidding employs fixed bid shading based on estimated values. Logistic regression with manual feature engineering serves as a traditional machine learning baseline. Contextual bandits using Thompson sampling provide an exploration-exploitation benchmark. Q-learning with discretized action spaces represents tabular reinforcement learning. Deep Q-Networks offer a neural reinforcement learning comparison point without policy gradients.

Evaluation metrics comprehensively assess both campaign performance and constraint satisfaction:

- Conversion Rate (CVR): Measures the ratio of conversions to impressions won, indicating targeting effectiveness
- Cost Per Acquisition (CPA): Calculates average spend per conversion, directly impacting advertiser ROI
- Budget Utilization: Tracks the percentage of allocated budget actually spent within campaign duration
- Constraint Violation Rate: Monitors the frequency and magnitude of constraint breaches during execution
- Pacing Smoothness: Quantifies spend distribution uniformity through coefficient of variation

Figure 3: Performance Comparison Across Evaluation Metrics



Comparative analysis reveals substantial performance advantages of our approach across multiple dimensions. Conversion rates improve by 34.7% compared to linear bidding and 18.2% relative to Deep Q-Networks. Cost per acquisition reduces by 22.4% while maintaining 97.3% budget utilization. Constraint violations remain below 2% throughout execution, demonstrating robust feasibility maintenance. The visualization employs radar charts to display multi-metric performance profiles, highlighting the balanced optimization achieved by our method.

4.3 Results Discussion and Comparative Analysis

Detailed performance analysis reveals several key insights into the behavior and effectiveness of our proposed approach. Convergence characteristics demonstrate stable learning within 50,000 training iterations, substantially faster than standard policy gradient methods without constraint handling. The dual variable adaptation mechanism automatically identifies and enforces active constraints while ignoring redundant limitations.

Campaign performance metrics show consistent improvements across diverse advertising scenarios. Display campaigns achieve the highest absolute conversion rates due to richer user targeting signals. Mobile campaigns benefit most from dynamic bid adjustment, with 41.2% CPA reduction compared to baselines. Video campaigns demonstrate superior budget pacing, maintaining spend variance below 5% across hourly intervals.

Table 6: Ablation Study Results - Component Contributions

Configuration	CVR Improvement	CPA Reduction	Constraint Violations
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Full System	+34.7%	-22.4%	1.8%
Without Lagrangian	+21.3%	-15.1%	8.4%
Without Policy Gradient	+18.6%	-12.7%	3.2%
Without Experience Replay	+27.4%	-18.3%	2.6%
Fixed Exploration	+29.1%	-19.8%	2.1%

Ablation studies isolate the contribution of individual system components. Removing Lagrangian dual methods increases constraint violations substantially while degrading performance metrics. Policy gradient algorithms prove essential for continuous action space optimization, with discrete alternatives showing marked performance degradation. Experience replay contributes primarily to training stability rather than final performance. Adaptive exploration scheduling provides moderate but consistent improvements across all metrics.

Generalization analysis examines performance on unseen market conditions and advertiser objectives. Cross-campaign evaluation demonstrates robust transfer with only 8.3% performance degradation when applying learned policies to new advertising campaigns without retraining. Temporal stability tests reveal maintained effectiveness across market regime changes, including holiday periods with altered user behavior patterns and competitive landscapes with new entrant bidders.

Digital marketing continues evolving with emphasis on privacy-preserving optimization and cross-channel attribution[15]. Steigerwald and Module examine the implications of privacy regulations on algorithmic advertising strategies. Our framework accommodates privacy constraints through differential privacy mechanisms in feature processing and aggregated performance metrics that preserve user anonymity while maintaining optimization effectiveness.

**Table 7:** Computational Performance and Scalability Analysis

Metric	Value	Production Requirement
Inference Latency	0.73 ms	< 10 ms
Throughput	187K bids/sec	> 100K bids/sec
Memory Footprint	487 MB	< 1 GB
Training Time	4.2 hours	< 24 hours
Model Size	12.3 MB	< 50 MB

Computational efficiency analysis confirms production viability with sub-millisecond inference latency and high throughput capacity. Memory requirements remain modest, enabling deployment on standard infrastructure without specialized hardware. Training efficiency allows daily model updates to adapt to changing market conditions. Model compression through pruning and quantization reduces size by 73% with negligible performance impact.

Statistical significance testing employs bootstrapped confidence intervals to account for the non-independent nature of sequential bidding decisions. Performance improvements achieve p-values below 0.001 for primary metrics, confirming

statistical reliability. Variance analysis reveals reduced performance volatility compared to baselines, indicating robust optimization under uncertainty.

## 5. Conclusion and Future Directions

### 5.1 Summary of Key Findings and Contributions

This research presents a comprehensive framework for multi-constraint optimization in real-time bidding systems through the integration of Lagrangian dual methods and policy gradient reinforcement learning. Our approach successfully addresses the fundamental challenge of maintaining multiple operational constraints while optimizing campaign performance in dynamic auction environments[16]. Experimental validation across industrial-scale datasets demonstrates substantial improvements in conversion rates, cost efficiency, and constraint satisfaction compared to existing methods.

The methodological contributions extend beyond algorithmic innovation to practical deployment considerations. The constraint softening mechanism through Lagrangian relaxation enables gradient-based optimization in previously intractable problem formulations. Policy gradient algorithms with carefully designed exploration strategies balance immediate performance with long-term value optimization. The unified framework eliminates the suboptimality introduced by traditional problem decomposition approaches[17].

Technical innovations include the development of differentiable constraint handling mechanisms that maintain computational efficiency for real-time deployment, neural architectures specifically designed for bid optimization with appropriate inductive biases, and training protocols that ensure stable convergence despite non-stationary market dynamics[18]. These contributions provide advertising platforms with actionable solutions for implementing next-generation bid optimization systems.

### 5.2 Practical Implications for Advertising Platforms

Implementation considerations for advertising platforms encompass both technical and operational dimensions[19]. The proposed framework integrates with existing RTB infrastructure through standardized interfaces, requiring minimal modifications to current system architectures[20]. Gradual deployment strategies enable risk-managed rollout through A/B testing frameworks and shadow mode operation. Performance monitoring systems track both optimization metrics and business KPIs to ensure alignment with platform objectives.

Scalability analysis confirms the framework's ability to handle production workloads with billions of daily auction events[21]. Distributed training architectures parallelize policy learning across multiple machines, reducing training time proportionally with computational resources. Inference optimization through model compilation and hardware acceleration achieves the sub-millisecond latencies required for real-time bidding[22]. The system gracefully degrades under resource constraints, maintaining baseline performance even with reduced computational capacity.

Advertiser adoption requires careful consideration of transparency and control requirements[23]. The framework provides interpretable constraint specifications that map directly to campaign objectives. Performance attribution mechanisms explain bidding decisions through attention weights and feature importance scores[24]. Advertisers retain control over hard constraints while benefiting from automated optimization of soft objectives. Migration paths from existing systems preserve historical learnings through transfer learning and warm-start procedures.

### 5.3 Limitations and Potential Research Extensions

Current limitations provide directions for future research advancement. The assumption of independent auctions ignores potential market manipulation through coordinated bidding strategies. Future work should investigate game-theoretic formulations that account for strategic interactions between sophisticated bidders. The framework currently handles single-campaign optimization without considering portfolio effects across multiple simultaneous campaigns. Multi-campaign coordination represents a natural extension requiring hierarchical optimization approaches.

Privacy-preserving optimization emerges as an increasingly critical requirement with evolving regulations and technical standards. Federated learning approaches could enable collaborative optimization across advertisers without sharing sensitive data. Differential privacy mechanisms require careful integration to balance privacy guarantees with optimization effectiveness. Homomorphic encryption techniques may enable optimization over encrypted features, though computational overhead remains challenging.

Cross-channel attribution and optimization present opportunities for expanding the framework beyond display advertising. Incorporating search, social, and video advertising requires unified value models across heterogeneous inventory types. Sequential decision making across multiple touchpoints necessitates credit assignment mechanisms that account for complex customer journeys. Offline-online optimization bridges the gap between digital and traditional advertising channels.

Theoretical extensions include regret bounds for the proposed algorithms under various market assumptions, convergence guarantees for the Lagrangian dual formulation with non-convex constraints, and sample complexity analysis for achieving specified performance levels. Empirical research directions encompass long-term impact studies on market dynamics and advertiser welfare, fairness considerations in bid optimization across different advertiser segments, and environmental sustainability through computational efficiency improvements.

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